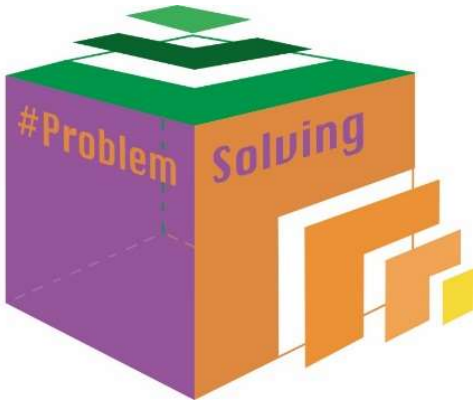
Teaching problem solving in technology rich environments

Teacher Guidance



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Why problem solving?

Problem solving and critical thinking are two of the key skills required in the world today. In this project, we focus on problem solving in the teaching of mathematics. The advantage you gain by learning and using problem solving in mathematics is the possibility to use it in your personal context/real life.

You have to consider the problem from different angles and choose a solution that includes most aspects and issues. The target is that the students go for their own ideas and tools in creative ways to solve the problems. Working in this way gives the students interesting and challenging tasks that are much more real than drilling and carrying out calculations stripped for context. Mathematics suddenly becomes a way of seeing and understanding the world instead of tasks where you have to make numbers change places.

Advice when you work with problem solving in mathematics

Set the scene - it is important that the students become aware of the context and the meaning of the problem.

Problem solving takes time - give the students time to think and struggle.

It is important that the students possess strategies to explore or solve the problems, which means that the problems you pose must not be more difficult than the students are able to handle. That does not mean that the problems must be simple, but they have to contain several possibilities of trying out different kinds of solutions.

It is important that the students during their struggle can get some hints (not answers) from their teachers, if they are stuck with a problem. The process belongs to the students.

The teacher must take the lead in scaffolding the lesson(s), because unexpected situations can easily arise during the lesson(s).

When the students are working on problems they need different kinds of concrete materials and useful technology, therefore you must provide these.

Working with problem solving can provide students with mathematical realizations, which are rich learning for the rest of the class. It is therefore important to pick up points during the lesson and give the students opportunities to share their knowledge and understanding with the class.

Students do not practise repetitive exercises with problem solving - they learn mathematics.

Students are motivated by relevant contexts in the problems; therefore, you should choose problems with contexts known by the students.

As a teacher, you are responsible for the students' understanding of the problem. Time must be spent to guarantee that the students are comfortable with the text in the problem.

Be aware that you do not search for *the answer* but focus on the process, the different ways to a possible solution and different solutions.

Teachers should explore the process with students by asking different kinds of questions about the process and the students' conclusions or reasoning.

Problem solving can support the students' curiosity for mathematics and different mathematical problems can encourage students to work in a free and investigative way.

An example presenting a problem:

Bettina loves glitter socks. She has three red pairs, two blue pairs, one silver, one green, one golden, four common black pairs and two pairs of tennis socks. All of her sock are lying in the drawer and are not put together in pairs. How many times does she have to draw two socks to get a pair of glitter socks that match? She is very conscious about her use of energy therefore; she has not turned on the light.

How do you recognise a good problem for your mathematics teaching?

You can find different categories of problems, and we have tried to give some of the characteristics in this list. You may not find all the characteristics in a single problem, but all problems will have several of the characteristics represented. Here are some of the most important characteristics.

A good problem:

- Has many different solutions or no solution
- Can be solved by students at different levels
- Contains relevant mathematics
- Promotes collaborative working
- Demands that the students plan their work
- Makes the students use their mathematical strategies
- Requires the students to justify their reasoning
- Gives the students competencies which they can use in another context
- Is differentiable

Example #1:

Which one is the odd one out?

3, 6, 9, 12

It is important that the students argue for their choices and that the teacher is positive to every suggestion from the students. You may hear some of the following suggestions:

3 - because it is the only prime number
6 - because it is a perfect number
12 - because it is the only two-digit number
9 - because it is a square number

You will hear a lot more suggestions - be positive to every suggestion ☺

Example #2

Old McDonalds farm.

At Old McDonalds farm, you find pigs and chickens. You can see 282 legs in all (you can pick any even number of legs, if you need to) - how many pigs and how many chickens does Old McDonald have?

Consider:
Is this a problem? Why is it/is it not a problem?
How can you scaffold your students in their solving of this problem?

Example #3

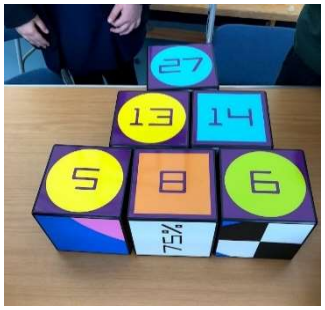
Anna has four boxes. She is weighing them, but she is a little crazy, therefore she chose to weigh the boxes in pairs. The first two boxes together weighed 6 kg. When she had finished weighing the boxes, she had written the different combinations for the pairs: 6 kg, 8 kg, 10 kg, 12 kg, 14 kg and 16 kg.
The next day she brings the boxes to school, and she tells what the boxes weigh in pairs. But what does each box weigh?
How do we figure out what each box weighs, when we cannot weigh each box for itself?

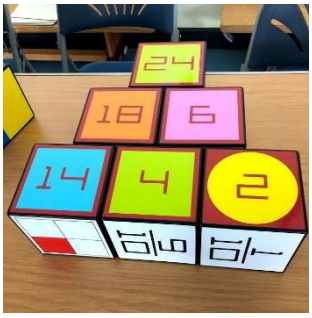
Solution: 1,3,5,9 or 2,4,6,10

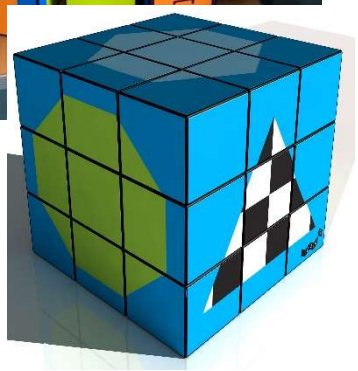
Example #4

If you have access to this cube, IZAK9, you can make different problems for your students:

a) You can pick a number, e.g. 27, and ask your students to build the number with some of the other cubes:







b) Different representations can be used as a way to visualize e.g. fractions
c) The pattern on the cube is a good task for a group of students to challenge their cooperative way of doing things

Working with problem solving

When you are working with problem solving it is essential to be strategic and systematic. The students do not automatically work this way, therefore it is important that you as a teacher are aware of this and scaffold the way the students can build up their problem solving strategies. The task below is an example from a Danish examination paper in December 2018:

The stars and the boxes in fig. 1 each weigh a number of grams (whole number)

a) Give three examples of how many grams the black weight can weigh. You must give a reason for your answer.

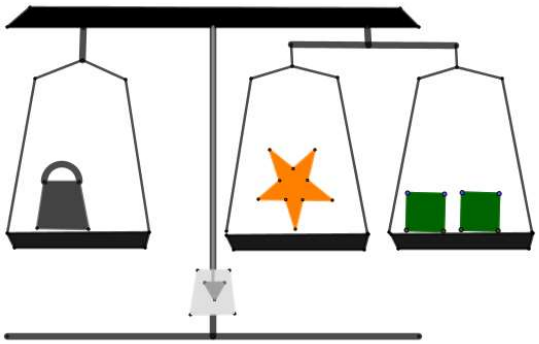


Fig.1

b) How many grams does a star weigh and how many gram does a box weigh on Fig.2?

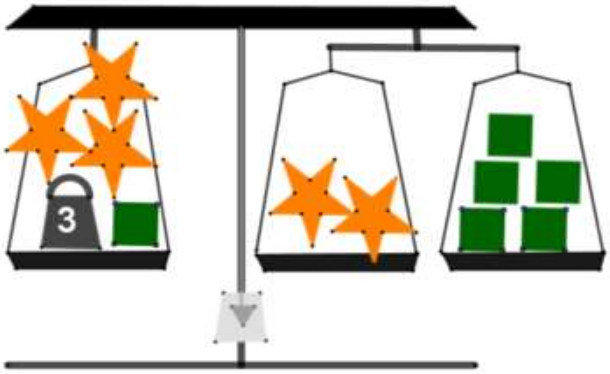


Fig.2

Drawings made in GGb by Kirsten Spahn

Consider:

Which signs do you see to reassure that it is a problem and not a task?
In what way do a) and b) differ as tasks/problems?

How do I support/scaffold my students? Moreover, how do I focus on the language?

When you are working with problem solving in mathematics you do not automatically follow a certain path or work towards one specific solution. The students must learn to formulate hypotheses themselves, to argue that a given hypothesis applies and can be proved. Mathematical sentences can be written this way: "If…then…", and mathematical arguments always contain "...because...". The students should be able to use these sentences effectively during problem solving and problem posing activities and thus demonstrate their mathematical thinking. If the problem presented to the students is the one about the four boxes and their weight, the teacher can support their mathematical thinking and argumentation using the following questions:

- If one of the boxes weighs…, what does one of the other boxes then weigh?
- Can you point out some numbers that the boxes cannot weigh?
- Do you find some numbers more appropriate than other? Is it possible to use decimal numbers?
- Is it possible that two of the boxes weigh just as much?
- If the boxes weigh a total of 6 kg, how much can each box possibly weigh?
- What happens if one of the boxes weighs 1 kg?
- How can you demonstrate that your solution is right?
- Can you explain how you found the system? You may use the following starters for your sentences: if two boxes weigh a total of 6 kg, then each box must weigh …kg and …kg or …kg and …kg, because it ends up being 6 kg in total
- Can you give a reason that these are the only solutions?
- Can you explain why all boxes weigh an even or an odd number of kg?
- Can you make a task similar to this one?

You may use different strategies to ensure that all students participate and are using the mathematical language in your teaching. The following strategies are useful:

- Peer to peer where the students have the opportunity to discuss solutions/explanations with each other before presenting their suggestions to the teacher during the class discussions. As a teacher you can plan how to scaffold the students (challenge them with support) to make their interaction more varied using e.g. 'why'cards and 'if…then…' cards.
- Different methods supporting interaction concerning mathematical subjects, e.g. groups of 'experts' where the students immerse themselves in a topic and communicate their findings to others

- Lessons with different tasks where the students move around in pairs, everybody is working all the time and the teacher is free to support the students requiring more support
- Attention to the planning of teacher instructions and structure in teaching. Students must be clear on what they are expected to learn and how they will achieve this
- Have patience, allow time for thinking and give students more time to answer on specific questions. This will enhance the students' own reflections
- Check-in at the beginning of the lesson when the student share their everyday experience on the subject of the lesson
- Check-out at the end of the lesson when the students share their learning from the lesson e.g. repeat a key learning point or explain a mathematical expression
- During the lesson the teacher picks up one or two key learning points to make sure that the students are on track

Working with problem solving and technology

The role of the teacher changes when the students are working with technology and problem solving. It is important that teachers guide the learning through active participation and effective questioning rather than overly prescriptive directing the use of mathematics or the technology. Problem solving demands a divergent and convergent way of thinking.

There is more than one answer to problems, which means that you may hear surprising and new kinds of answers from the students. You may not have anticipated some of the approaches that the students will use to find an answer! Afterwards, it is important that the teacher guides the students to pose open-ended questions that lead the students to find appropriate solutions to the problems.

Questions that teachers might ask the students

A problem solving task has many solutions and strategies. The teacher should explore the working process, strategies and considerations about solving the problem with the students. It is also the teachers' job to guide the students through the process if they meet challenges. You promote learning by using some of the following questions:

- Which strategy did you use?
- Can you think of another way to find a solution?
- Could you change the numbers and get new knowledge?
- Can you explain……..?
- What do you think…..?
- Why……?
- Is there an easier way……?
- Do you know a digital or analogue technology that might help you?
- What do you know now?

Four important competencies

Competencies are central in the work with technology and problem solving in mathematics teaching.[1]

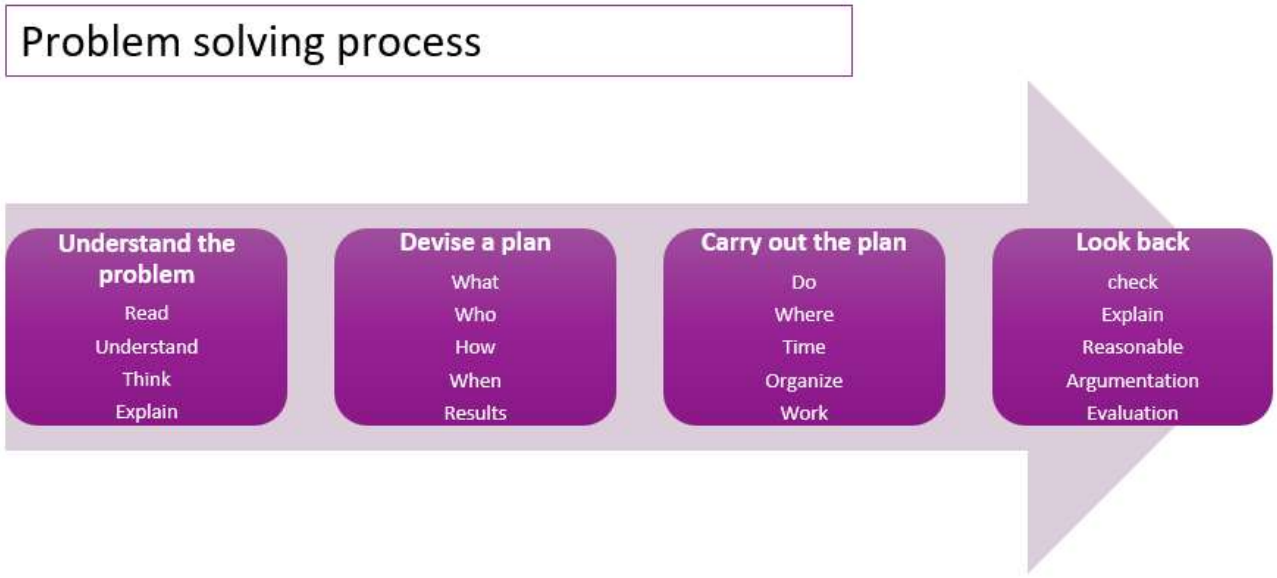
Thinking mathematically (mastering mathematical modes of thought) is a central competence and it shows the students' capability of posing questions and giving answers related to their problem solving. The students' computational thinking is also an important element in the process of finding solutions, where their understanding for the technology can support their different opportunities for finding solutions.

Posing and solving mathematical problems is the competency where the students show their abilities making them able to find a solution. Using this competency the students are able to plan and structure their working progress that gives them the possibility to find solutions to a specific task

Modelling mathematically (i.e. analysing and building models) gives the students tools that support their way of analysing and build mathematic models that demands more than standard calculations to find solutions.

The last relevant competency is *Reasoning mathematically*, which makes the students able to think mathematically and judge the durability of the mathematical allegations they present.

Guide for students working with problem solving:



[1] http://www.math.chalmers.se/Math/Grundutb/CTH/mve375/1112/docs/KOMkompetenser.pdf pages 7-8

Teaching Problem Solving In Vocational Schools

The problem solving tasks in vocational school depend on the field of studies taught. In the Finnish curriculum, the teaching of mathematics in vocational school is the same for every field but the tasks are varied. In the new curriculum, it is said that students learn how to solve mathematical problems related to their vocational field and to everyday life.

Nowadays, in working life, employers respect more and more workers' skills in problem solving and especially in technology rich environments. The employee is expected to solve problems in technology rich environments, not just mathematically. In vocational school, it is important to learn mathematics as mathematics is used as a tool in solving problems in working life. The ability to solve professional and practical tasks can be developed by practising mathematical problem solving tasks. Various strategies and models of thinking are learned by doing problem solving tasks. In addition, the ability to work out even more challenging problems increases. Ability to think mathematically is developed and diversified. Students are learning to see and use different options. It is also good to know how to do problem solving in technology rich environments.

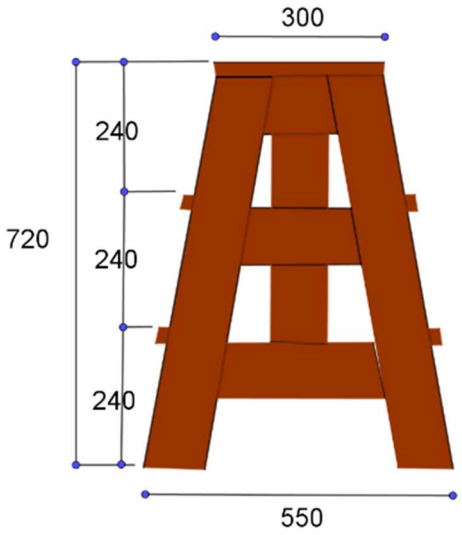
In Finnish vocational schools, mathematics is sometimes taught by a vocational teacher instead of a teacher of mathematics. The benefit of this is that the vocational teacher knows and understands the field of studies better than the teacher of mathematics. Vocational teacher knows what the requirements of working life. The downside to this is that the vocational teacher is not necessarily familiar with the pedagogy for teaching mathematics. Hämäläinen et. al. (2015) concluded: "Vocationally trained adults must be able to quickly take over new technologies and to solve problems in changing technological environments."

Sources: Hämäläinen, R., Wever, B., Malin, A., & Cincinnato, S. (2015). Education and working life: VET adults' problem-solving skills in technology-rich environments. *Computers and Education*, 88 (October)

Sample problems for vocational students:

Problem solving tasks for technical student could be like this.

The wooden stool is made of 20x100 wooden planks. How long are the support planks?



Plan steps you can easily clime. The threshold is at 75 cm above the ground and the distance from the door is 150 cm.

Problem solving tasks for cooks could be like this.

The executive chef wanted to know, whether it is more economical to use peeled or unpeeled carrots in the kitchen. Peeled carrots cost 1.23 €/kg and unpeeled carrots cost 0.60 €/kg. The share of peels is 16 % of the weight of carrots. A good chef peels about 12 kg carrots in one hour. The cost of labour for one hour is 17 €. Which alternative is **better**?

Problem solving task for students who are studying cleaning services

You have 3 litres of 5% solution and you want to dilute it to a 2% solution. How much water do you need to add?

Teaching Problem Solving in an Adult Education Setting

Problem solving approaches in adult education are very similar to those in vocational and school education, they are best suited in a relevant and everyday context. There is often no fixed curriculum in adult education settings, adults choose to learn subjects that are useful to their lives or of interest to them. For example, they might choose to study numeracy or mathematics to help them progress in their work, help their children with homework or to manage their household budget. As such, tutors develop the courses with their learners' needs and interests at the centre of what they teach. Problem solving in adult education is one of many approaches used to understand whatever subject the adults are studying.

Tutors who teach adults may have some training in tutoring and will be confident in their numeracy and mathematics abilities but will not be trained as mathematics teachers. They will have to draw on their own knowledge and skills to create a curriculum that incorporates a problem solving approach or attend professional learning to learn how to use problem solving approaches in their work.

Adult learners will often study numeracy in a context to make it more meaningful and interesting. For example, adults may learn numeracy by learning about the environment, weather and climate and looking at the numeracy we can use to understand these subjects. In a group where the tutor does not take a problem solving approach, adults studying environmental topics such as use of energy or looking at climate change might be asked to read information from tables and discuss information provided for them through tutor input. In a group where the tutor adopts a problem solving approach, the tutor might start the session by introducing the topic of weather and climate and then go on to set problems for learners. For example, learners could be asked to look at the sky outside and, alongside their fellow learners, devise a means of measuring how much cloud cover there was at that time. In this instance they are not performing routine calculations but, with others in the group, they are asked to solve the problem of how they might make a reasonable measurement of cloud cover.

Alternatively, if they are studying numeracy with the aim of gaining access to college to study to become a painter and decorator, the tutor who does not take a problem solving approach may provide the measurements of a room and the costs of paint per tin and ask students to provide the costs for the paint for the room. A tutor who is taking a problem solving approach may not provide any measurements or tools but ask learners to solve the problem of how they can work out how much paint is required and how much it will cost. There will be a number of approaches and a number of answers depending on whether the results are estimated or calculated. This provides a real-life problem solving scenario.