Teaching problem solving in technology rich environments

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Literature Review

Introduction

In the current climate of globalisation and international development, problem-solving has been deemed a necessary skill, which students require to fulfil their potential in future employment, in further learning situations, and as global and national citizens (OECD, 2004). The opinion has developed that the world needs problem-solvers who will be able to adapt to our continually changing societies (Guven & Cabakcor, 2013), and this has been reflected in international curricular reform over the last two decades (Guven & Cabakcor, 2013; McDonald, 2017; Prendergast et al., 2018; Schoenfeld, 1992; Soh, 2008).

With this is mind, the following review of the literature on the subject seeks to clarify the notion of problem-solving itself, its importance, as well as a number of issues regarding its implementation. Initially, a discussion on the various definitions of problem-solving will take place, followed by a description of the importance of problem-solving in a mathematical context. At this point, the practicalities of implementing problem-solving will be addressed, discussing both barriers and supports to effective problem-solving in a classroom situation.

Finally, the related topics of technology in the classroom and problem-posing will be analysed with regard to promoting and developing problem-solving in the mathematics classroom. The review of literature focuses mainly on research that has taken place within second level (upper and lower) mathematics education settings. While this is a cross sectoral project, there was a dearth of literature available from other sectors. It is important to keep this in mind when reading the review and to also consider the implication and significance of the findings for the primary, tertiary and adult education sectors.

What is problem solving?

Problem-solving has been a primary topic of study in international research these past forty to fifty years. In spite of this, a common definition amongst researchers and educators remains elusive – particularly when it comes to the specifics of *mathematical* problem solving (Lester, 1994; McDonald, 2017; Schoenfeld, 1992; Stanic & Kilpatrick, 1988; Wilson, Fernandez, & Hadaway, 1993). Indeed, Gugnetti & Jaquet (2005), as cited by McDonald (2017), go so far as to suggest that a universal definition of problem-solving simply does not exist.

Regarding the difficulties in finding a suitable definition, Schoenfeld (1992) points out that in society, multiple contradictory meanings of the word 'problem' exist; the spectrum runs from viewing a problem as any mathematical exercise, to problem-solving as critical thinking, and as a way to address the big questions at the heart of mathematics. He points out that this is reflected even in the dictionary definition of the term, which remains true today. The Oxford English Dictionary (2018) provides a variety of definitions for "problem" including:

1. A difficult or demanding question
2. *Math*. and *Science*. A proposition in which a specified action is required to be done... Later also: a phenomenon or situation to be physically explained or mathematically described or solved.

Under the same entry "problem-solving" is defined as the action of finding solutions to difficult or complex issues.

In spite of this lack of clarity and rigour in defining a problem, there are a number of key ideals which have presented themselves thematically in mathematics education research. The first is a problem as a complex question to which there is no obvious solution – i.e. we can only call something a problem if we don't know how to solve it immediately (e.g. Schoenfeld, 1983). As such, distinct from a simple mathematical exercise, where a rote-learned method will lead the student to a solution, a problem-solving activity requires students to use their knowledge in new ways, and often to try a multitude of different approaches. Secondly, this viewpoint implies that a certain creativity can be implemented and nurtured in an act of true problem-solving (Silver, 1997). Singer, Ellerton, and Cai (2013) touch on this when noting that "problem-solving is meaningful if it engenders invention or at least reveals new issues" (p. 1). Taking this into account, is also important to note the relationship between the problem and the individual – a problem for one person is not always a problem for another (Wilson et al., 1993). This must be kept in mind when dealing with problem-solving in an educational context.

Beyond all of this, it has been argued that problem-solving lies at the heart of mathematics and is the basis of the work of a mathematician (Schoenfeld, 1983; Halmos, 1980, as cited in Schoenfeld, 1992; Wilson et al., 1993). Schoenfeld (1983, p 11.) said of mathematics students: "they have no idea that 'understanding' mathematics means asking questions until things make sense". He argues that developing their problem-solving skills is the only way to give them a window into the world of the mathematician; to allow them to probe and question is to help them to experience true mathematics.

Importance of problem solving in mathematics

As previously mentioned, there are many who believe that problem-solving is not only *important* to mathematics, but that it is at the heart of all mathematical endeavour (McDonald, 2017; Schoenfeld, 1992; Stanic & Kilpatrick, 1988; Wilson et al., 1993). From this point of view, the importance of problem solving in mathematics, or more specifically of implementing problem-solving in mathematics education, cannot be underestimated. Ernest (1991) states that the aim of mathematics is to empower learners. As such, in helping students to see mathematics as an activity dealing with practical and theoretical problems of the world, educators empower them to become mathematicians and problem-solvers of tomorrow (Schoenfeld, 1992).

Analysing this more specifically, Lambdin (2003) argues that problem-solving helps develop a depth of mathematical knowledge otherwise unattainable. As summarised by Anderson (2008, p.2):

> As students are challenged by problem-solving tasks, it becomes necessary for them to think about what they already know; to consider strategies that may help them solve the problem; to engage in discussions with other students and the teacher in an effort to resolve conflicts; and to ultimately reorganise existing knowledge to accommodate new ideas

The depth of learning which takes place in such a situation cannot be compared to learning that consists of the application of rules or implementing a tried and tested method. Furthermore, the skills which are learned in problem-solving situations are ones which are widely applicable, and beneficial to students in life long after the finish of their mathematics education (Schoenfeld, 1983).

This is reflected in the current prevalence of problem-solving in curricular reform worldwide. It should be noted that a focus on problem-solving in mathematics education is a relatively recent phenomenon, with widespread research in the area taking place from roughly the 1970s and 1980s (Lester, 1994; Schoenfeld, 1992; Wilson et al., 1993). However, since then, there has been international recognition of its importance. This is evidenced in the new focus on problem-solving in U.S curricula as of the 1980s, as well as a definitive shift towards problem-solving in national curricula in countries such as Scotland, Ireland, Turkey, and Singapore (Guven & Cabakcor, 2013; McDonald, 2017; Prendergast et al., 2018; Schoenfeld, 1992; Soh, 2008). It is evident that in the current era of globalisation, countries worldwide are recognising the need for critical thinkers adaptable to the changing world. Problem-solving is being internationally recognised as the way to achieve this (Guven & Cabakcor, 2013).

Barriers to effective problem solving in the mathematics classroom

In spite of the widespread regard for problem-solving in the mathematics education community (Lester, 1994; Schoenfeld, 1992; Wilson et al., 1993), as well as the largescale appearance of problem-solving on national curricula in recent years (Guven & Cabakcor, 2013; McDonald, 2017; Prendergast et al., 2018; Schoenfeld, 1992; Soh, 2008), problem-solving in the mathematics classroom does not always occur to the extent that one might expect. Anderson, Sullivan, and White (2004) point out that in the Australian context, while teachers generally report that they endorse and support the prevalence of problem-solving on the curriculum, this is often not reflected in their actions in the classroom. There are a number of possible reasons for this, which will be discussed in terms of barriers to effective problem-solving for teachers and students.

Initially, from a teacher's point of view, a wide range of factors can create difficulties in the implementation of problem solving in the classroom. First of all, teacher beliefs and a school teaching culture can play a huge role in facilitating problem-solving in the classroom successfully (Thompson, 1992, as cited in Anderson et al., 2004). Those who hold more traditionalist teaching beliefs (with behaviourism and abstraction being seen as important in mathematics learning) tend to be less likely to implement problem-solving in the classroom than those who hold more contemporary, social-constructivist beliefs (Anderson et al., 2004). Furthermore, Hoyles (1992) points out that these beliefs must be viewed in context, as there are many external factors which can affect teacher beliefs, and thus teacher practice. Frequently cited amongst these are issues of time, with teachers struggling to implement time-consuming problem-solving activities under the constraints of their education system (Anderson et al., 2004; Little & Anderson, 2016; Prendergast et al., 2018). Similarly, the point has been made that problem-solving is much more difficult for teachers to assess than other more traditional learning activities (Jones, Swan, & Pollitt, 2015; Wilson et al., 1993). Furthermore, it has been found that several teachers hold the attitude that problem-solving can only be carried out with higher ability students, and only at certain grade levels (Anderson et al., 2004; Little & Anderson, 2016).

Concomitantly, student-based barriers to problem-solving implementation also exist, mainly in the form of student attitudes and beliefs. These include entrenched views that problem-solving is purely about getting an answer and should be simple (Wilson et al., 1993), and frustration and rejection of ideas that do not follow a traditional text-book approach (Anderson, 2008). Furthermore, Schoenfeld (1992) points out that many students are held back by long-held beliefs that mathematical problems not only have just one right answer, but also only one right method to attain this answer. They believe that if you have studied a topic this method should be obvious. Added to all of this are issues of student anxiety and self-efficacy in problem-solving (Guven & Cabakcor, 2013). It is thus evident that implementing effective problem-solving in a school context is no simple task.

Supports to enable effective problem solving in the mathematics classroom

Bridging the gap between research and practice in the mathematics classroom can be a difficult goal to achieve, as previously discussed. More specifically it is also important to bridge the gap between the literature on improving teaching and learning and the literature on policy and leadership in schools (Cobb, Jackson, Henrick, & Smith, 2018). Anderson (2008) points out that in order to develop successful problem-solvers in the classroom, the teacher must grapple against various constraints, and aim to discuss the importance of problem-solving with their students, to use rich problem-solving tasks, and to prioritise higher-order questioning techniques. Furthermore, they must allocate sufficient time for these tasks to be effective. In order to overcome the cultural and practical difficulties in implementing this, some supports must be provided.

The first and perhaps most obvious support is continued professional development for in-service teachers (Guven & Cabakcor, 2013). Furthermore, as a move to support both pre-service and in-service teachers in the implementation of problem-solving, Little and Anderson (2016) suggest strong ties being built between schools and research institutions, building up a body of cooperating teachers who would work both with researchers and pre-service teachers in the school. This would help to create a contemporary mathematics education culture in schools, and also allow teachers on the ground to voice their concerns and provide feedback to research institutions.

In addition, Cai, Morris, Hohensee, Hwang, Robinson, and Hiebert (2017) suggest that in order to ease pressure for teachers, a shared and regularly updated online database of knowledge and resources would be invaluable. Access to resources was also suggested as a time-saving support by pre-service teachers in the Australian study carried out by Little and Anderson (2016). Furthermore, Jones et al. (2015) suggest what is known as the comparative judgement technique for assessment of problem-solving tasks. Using this method, a community of practice compare students' work and create a scale, considering the work of every student before clarifying grades. This is in order to avoid the subjectivity which can occur is assessing problem-solving. In the same study (which took place in the United Kingdom), it was found that teachers were much more comfortable using comparative judgement techniques to grade problem-solving tasks than traditional marking schemes in line with high-stakes assessments.

These supports, amongst others, may help in creating a community of practice which supports problem-solving in mathematics classrooms. As pointed out by Schoenfeld (1992), mathematics learning is culturally shaped and defined. As such, the development of a strong community of practice is extremely important in creating the social environment necessary to support problem-solving in the mathematics classroom.

The role of technology in promoting problem solving in the mathematics classroom

Alongside the push for problem-solving in current mathematics education circles, much research has also taken place regarding technology in the mathematics classroom. It has been found that digital tools can pave the way for inquiry-based, collaborative, realistic mathematics learning (Bray & Tangney, 2017). As such, an obvious link exists between the two research areas, and technology has been cited by many as a means of promoting and enhancing problem-solving activities in the mathematics classroom (Bray & Tangney, 2014; Bray & Tangney, 2017; Jacinto & Carreira, 2014; Lopez-Real & Lee, 2006). Indeed, in a recent review on the use of technology in mathematics teaching, it was stated that "the use of digital technologies that align with a more constructivist, epistemic approach may have the capacity to ... [facilitate] realistic, problem-solving and collaborative approaches to teaching and learning, and [provide] coherency and context for mathematics" (Bray & Tangney, 2017, p.258).

Lopez-Real and Lee (2006) highlight that when discussing technology with regard to problem-solving, it is important to see technology not only as an environment in which to learn mathematics, but as a tool for problem-solving in itself, which may provide new and diverse methods for solving problems. They point out that in many cases, the use of ICTs is an obvious, or natural step in solving certain mathematical problems (using geometry software for geometrical construction problems for example), but that even for more abstract problems, the application of technology can be enlightening for all. Bray and Tangney (2014) expand on this further, highlighting the importance of technology in bringing meaningful, collaborative mathematics to the classroom, and in implementing social constructivist techniques often in line with problem-solving ideals. Importantly, they specifically highlight the benefits found in improving attitudes and engagement through the use of digital technology. In their study, students reported enjoying the technological approach for the independence and variety it provided, as well as the manner in which they were pushed to solve problems.

It should be noted that, as with other aspects of problem-solving, effective implementation of ICTs requires pedagogical transformation, and an educational culture change (Bray & Tangney, 2014; Bray & Tangney, 2017). In order for use of technology to be properly implemented (providing a transformed or at least augmented learning experience for students) longer class periods are required, as well as the implementation of contemporary pedagogical approaches that see the teacher as facilitator and allow for collaborative, independent work amongst the students (Brendan & Tangney, 2017). Only in this way technology can be effectively used to enhance mathematical problem-solving.

Problem posing – the forgotten ally?

Another idea which goes alongside mathematical problem-solving is that of problem-posing. However, until recently, it has often been given less attention than problem-solving in international research, as well as in curriculum documents and textbooks (Cai and Jiang, 2017; Singer et al., 2013; Wilson et al., 1993). Nevertheless, research in this area is growing, and the advantages of implementing problem-posing in mathematics education have been put forward by several in the field (Silver, 1994; Silver, 1997; Singer et al., 2013; Voica & Singer, 2011).

To begin with an explanation, problem-posing as described by Silver (1994) consists of both the generation of new problems, and the reformulation of problems already given. As such, in relation to problem-solving, problem-posing can occur before, during, or after the solution to a problem is found. An example of a problem-posing activity is the What-if-Not idea, where students are encouraged to change the attributes of a given problem to formulate a new one, and to explore the possibility of finding a solution (Brown & Walter, 1983, as cited by Wilson et al., 1992). Singer et al. (2013) point out that in today's dynamic world, school leavers are required to be flexible, and to adapt to unpredictable situations; they suggest problem-posing (alongside problem-solving) as a means of preparing students for this eventuality.

The advantages of implementing problem-posing into mathematics instruction are many. First of all, it falls in line with the collaborative, social-constructivist approaches often associated with problem-solving, and thus caters for the development of communication skills, independent learning, and creativity (Singer et al., 2013). Silver (1997) also points out the importance of problem-posing in developing creativity, and in addition, makes the case that problem-posing can be a useful tool in the *assessment* of creativity. Speaking in broader terms, like problem-solving, problem-posing has been cited as central to the work of mathematics (Silver, 1994). As such, it acts as a window to the understanding of mathematics for students, helping them to mathematise the world around them, and apply mathematics to real situations. Moreover, in connecting mathematics to students' interests, and encouraging peer collaboration, it has also been shown to improve student attitudes (Silver, 1994).

In spite of the many advantages of this approach, as well as the recent appearance of problem-posing in some curricular reforms, it has been found that the implementation of problem-posing in textbooks and other curricular documents remains much lower than that of problem-solving (Cai & Jiang, 2017). Given the benefits of problem-posing both in its own right and in relation to problem-solving, the importance of continual emphasis on the idea in the mathematics community cannot be underestimated.

Conclusion

Throughout this review, a discussion has taken place on the meaning and importance of problem-solving in the society of today. The difficulties of timing, assessment, and culture in implementing problem-solving in the classroom have been addressed, as well as supports through means of professional development, and supply of resources with the aim of building a strong community of practice. The place of technology and problem-posing in promoting problem-solving in the classroom were also discussed.

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